Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Student number\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Assignment 1**

Derive the virtual work expression  of the bar element (length *h*) if the approximation is linear (a two-node element) and , *G* and  are constants.

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*G,J*

**

**

*x*

*h*

**Solution template**

Virtual work densities of the internal and external forces of the torsion bar model are

 and ,

in which  is the second moment of area with respect to *x*-axis, *G* is the shear modulus, and  is the external moment per unit length.

Let us start with the linear approximation to the rotation angle. The origin of the material coordinate system is at node 1 and the length of the bar is *h*.

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When the approximation is substituted there, virtual works of internal and external forces per unit take the forms

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Virtual work expressions are integrals of the densities over the length. Then, virtual work expressions of the bar element are given by

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